Vortex-induced vibrations of streamlined single box girder bridge decks

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ABSTRACT: Vortex-induced vibrations may occur on slender structures such as chimneys, towers, and bridge decks. Experience has shown that vortex-induced vibrations become more and more important for cable-supported bridges with increasing span. Examples of full-scale vibrations originating from vortex shedding are Great Belt Suspension Bridge in Denmark just before completion and Osterøy Suspension Bridge in Norway.

Three streamlined steel box girders have been investigated with two main focus areas: Influence of the bottom slope turning angle and Reynolds number. The box girders have been tested in low turbulent flow on scale 1:50 and 1:10 section models suspended in a dynamic as well as in a nominally non-vibrating test rig.

The Strouhal numbers determined in the dynamic test rig are consistently less than the Strouhal numbers based on the results found in the nominally non-vibrating test rig. The vortex-induced vibrations measured in the dynamic test rig depend strongly on the Reynolds number. Low Reynolds numbers for the 1:50 scale model give much larger vibrations than found for intermediate Reynolds numbers simulated with the 1:10 scale model. This behaviour is also found for circular cylinders.

Theoretical fit of the vortex-induced vibration amplitudes based on the spectral model, Eurocode approach 2, is in good agreement with the measurements carried out.

KEY WORDS: Vortex-induced vibrations; Strouhal number; streamlined single box girder bridge decks; wind tunnel tests; Reynolds number; bridge deck remedies; Eurocode.

1 INTRODUCTION

Vortex-induced vibrations may occur on slender structures such as chimneys, towers, and bridge decks. Experience has shown that vortex-induced vibrations become more and more important for cable-supported bridges with increasing span. Examples of full-scale vibrations originating from vortex shedding are Great Belt Suspension Bridge in Denmark just before completion [13] and Osterøy Suspension Bridge in Norway [14].

The increased spans of cable-supported bridges constructed over the last decades have resulted in several examples of vortex-induced vibrations of bridge decks, and today’s cable-supported bridge design has reached a structural detailing, where connections (i.e. bearings and hangers) are constructed with less friction reducing maintenance. The focus on maintenance and a “clean” structural detailing is not beneficial for damping.

The fact that vortex-induced vibrations are very sensitive to structural damping clearly show the need for clear design procedures focusing on methods to predict this type of vibrations. The vortex-induced vibrations may not be important for ultimate limit state designs, but the serviceability requirements should specify maximum vibration amplitudes for typical wind velocities relevant for vortex-induced vibrations.

A full analytical description of vortex-induced vibrations is still not available, and the procedures used to predict vortex-induced vibrations of structures are still rather crude. The cross-wind forcing mechanisms have proved to be so complex that there is no general analytical method available to calculate cross-wind structural response. The main physical parameters involved in the forcing mechanisms have been clarified, but the basic data used in full-scale predictions have not reached a general agreement among researchers. Especially the methods used to take aeroelastic effects, i.e. motion-induced wind loads, into account differ considerably. The lack of a common understanding is reflected in the Eurocode on wind actions [18], where two basically different calculation methods are specified.

The calculation procedure focused on in this paper is based on the spectral model originally suggested by [3], further refined by [4], extended to rectangular cross sections in [7], and used in Eurocode approach 2. [1], [3], and [4] mainly include circular cross sections. The design procedure presented in this paper extends the scope to streamlined single box girder bridge decks, and wind tunnel tests in low turbulent flow are used to estimate the parameters of the calculation procedure. The influence of the Reynolds number is investigated by testing the streamlined decks on different scales and at several wind velocities.
The paper furthermore presents remedies for vortex shedding based on wind tunnel tests of Hardanger Suspension Bridge in Norway.

2 FULL-SCALE EXPERIENCE

Since 1992 nine Norwegian suspension bridges have been constructed with a streamlined steel box girder: Askøy (1992), Gjemnessund (1992), Lysefjord (1996), Osterøy (1997), Storda (2001), Bømla (2001), Fedafjord (2006), Dalsfjord (2013), and Hardanger Suspension Bridge (2013). The locations of the Norwegian suspension bridges may be seen in Figure 1. These are typical one span suspension bridges, where the span length varies from 331 m to 1,310 m, and shape of the box girder cross sections resemble model 1c in Figure 5. Table 1 shows the properties of the mentioned Norwegian bridges as well as Great Belt Suspension Bridge (1998) and Little Belt Suspension Bridge (1970) in Denmark.

![Figure 1. Location of Norwegian suspension bridges with a streamlined steel box girder.](image)

<table>
<thead>
<tr>
<th>Bridge</th>
<th>Main span [m]</th>
<th>In-wind width, $b$ [m]</th>
<th>Cross-wind height, $h$ [m]</th>
<th>Mass, $m_e$ [t/m]</th>
<th>Damping, $\delta_e$ [%LD]</th>
<th>$S_{CQ}$ (eq. (7)) [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Askøy</td>
<td>850</td>
<td>15.5</td>
<td>3.0</td>
<td>9.5</td>
<td>2.0*</td>
<td>6.5</td>
</tr>
<tr>
<td>Gjemnessund</td>
<td>623</td>
<td>13.2</td>
<td>2.6</td>
<td>6.4</td>
<td>2.0*</td>
<td>6.0</td>
</tr>
<tr>
<td>Lysefjord</td>
<td>446</td>
<td>12.3</td>
<td>2.7</td>
<td>6.0</td>
<td>2.0*</td>
<td>5.8</td>
</tr>
<tr>
<td>Osterøy</td>
<td>595</td>
<td>13.6</td>
<td>2.6</td>
<td>7.4</td>
<td>2.0*</td>
<td>6.7</td>
</tr>
<tr>
<td>Storda</td>
<td>677</td>
<td>13.5</td>
<td>2.7</td>
<td>7.0</td>
<td>2.0*</td>
<td>6.1</td>
</tr>
<tr>
<td>Bømla</td>
<td>577</td>
<td>13.0</td>
<td>2.6</td>
<td>6.5</td>
<td>2.0*</td>
<td>6.2</td>
</tr>
<tr>
<td>Fedafjord</td>
<td>331</td>
<td>13.6</td>
<td>2.6</td>
<td>7.3</td>
<td>2.0*</td>
<td>6.6</td>
</tr>
<tr>
<td>Dalsfjord</td>
<td>626</td>
<td>12.9</td>
<td>2.5</td>
<td>7.5</td>
<td>2.0*</td>
<td>7.4</td>
</tr>
<tr>
<td>Hardanger</td>
<td>1,310</td>
<td>18.3</td>
<td>3.3</td>
<td>12.5</td>
<td>2.0*</td>
<td>6.6</td>
</tr>
<tr>
<td>Little Belt (DK)</td>
<td>600</td>
<td>26.6</td>
<td>3.0</td>
<td>11.7</td>
<td>2.0*</td>
<td>4.7</td>
</tr>
<tr>
<td>Great Belt (DK)</td>
<td>1,624</td>
<td>31.0</td>
<td>4.0</td>
<td>19.0</td>
<td>1.0 - 2.0*</td>
<td>2.5 - 4.9</td>
</tr>
</tbody>
</table>

*The logarithmic damping decrement is assumed to be 2% based on full-scale measurements on Askøy Suspension Bridge [17].
**The logarithmic damping decrement is based on full-scale measurements on Great Belt Suspension Bridge [13].

A few months before the opening, the Great Belt Suspension Bridge on several occasions experienced relatively large amplitude harmonic oscillations in vertical modes caused by vortex shedding for wind perpendicular to the bridge and in the...
range of 4 m/s to 12 m/s. To suppress the vortex-induced vibrations, guide vanes were installed beneath the bridge section along the main span.

The wind flow passing a bluff body like a bridge box girder has a tendency to separate at sharp corners, and to form a wake of vortex formations on the leeward side of the box girder. Therefore, on the surface of a bluff body there will always exist regions of vorticity where the flow is more or less coherent. The size of these regions and their impact on the loading depends on the Reynolds number.

In the wake of a bridge box girder there will develop a more or less regular vortex trail due to periodic shedding of vortices on the body. The shedding frequency depends on the Strouhal number, which depends on the shape of the cross section. Lock-in occurs when the mean wind velocity is such that the shedding frequency is close to the natural frequency of cross-wind vibrations. In the ensuing mean wind velocity lock-in range, the shedding frequency will not fulfill the equation \( n_s = St \nu / h \), but remain “locked on” to the relevant natural frequency of the structure. A well-known phenomenon of vortex-induced vibrations is the considerable time it takes to build up displacement response and reaching the point of lock-in.

Since the 1960s wind tunnel tests of streamlined steel box girder section models discovered that these box girders were susceptible to vortex-induced vibrations, see for instance the results obtained for the Little Belt Suspension Bridge [11]. Remedies like guide vanes to suppress vortex-induced vibrations were developed, and the bridges were constructed on the basis of installment of guide vanes if necessary. Four bridges needed urgent support of guide vanes – i.e. Great Belt, Osterøy, Bomla and Storda. Hardanger and Dalsfjord are exceptions because guide vanes and a vortex spoiler were included in their design. Guide vanes were also included in design of Little Belt and Fedafjord. After 20 years in service, guide vanes were installed at Gjemnessund. Then, two of the bridges included in Table 1 remain without remedies for suppression of vortex shedding vibrations: Askøy and Lysefjord.

3 SCALING EFFECTS

Generally, it is expected that loads induced by a fluid flow around a body are determined by the established flow field around the body. The characterization of the flow field is the forces inside the fluid, such as the friction and inertia forces, acting on the surface of the body. If the ratio between all the acting forces are identical in both full scale and model scale, the two flows may be considered being similar to each other, meaning that full physical similarity is achieved. However, in practice it is not necessary to match every full-scale force to the model. When basing the similarity on e.g. Reynolds number, fluid mechanical similarity is achieved. This covers the majority of physical phenomena in connection to flow-induced load and responses.

In general, fluid mechanical similarity is not possible to simulate in wind tunnel testing as the physical properties of the air are identical in both full scale and wind tunnel. Likewise, the gravity of the earth is a natural constant and invariable through all scales. This means only partial similarity is achieved allowing a certain degree of scaling mismatch on secondary flow phenomena.

Another criterion is the similarity of the flow field geometry and acting forces in full scale and in model scale. This assumption is based on the fact that a specific unique ratio of all acting forces will create a corresponding specific unique flow field, giving an identical flow pattern and hence identical force ratios. Analyzing the geometry of a flow field, it is obvious that in model scale, two related flow fields need to be taken care of, namely the background and the near flow around the body. The properties of the background flow are generally described by the airflow distribution of mean air velocity, turbulence intensity, coherence and integral length scale of the turbulence, while the near field of the flow around a body is characterized by stagnation, separation and reattachment of the flow on the body. Typically, one can assume that the background flow is properly scaled, meaning the near flow around the body needs proper scaling. This can be done by Reynolds law, which specifies identical ratios between inertial and friction forces in model scale (MS) and in full scale (FS) as shown in Equation (1):

\[
\frac{F_{I,\text{MS}}}{F_{\mu,\text{FS}}} = \frac{1/2 \rho_{\text{MS}} v^2_{\text{MS}} A_{\text{MS}}}{\mu_{\text{MS}} \frac{\partial v_{\text{MS}}}{\partial y_{\text{MS}}}} \quad \frac{1}{\frac{1}{2} \rho_{\text{FS}} v^2_{\text{FS}} A_{\text{FS}}} \quad \frac{\partial v_{\text{FS}}}{\partial y_{\text{FS}}} \quad \frac{v_{\text{MS}} L_{\text{MS}}}{v_{\text{FS}} L_{\text{FS}}} \rightarrow \frac{v_{\text{MS}}}{v_{\text{FS}}} \rightarrow \text{Re}_{\text{MS}} = \text{Re}_{\text{FS}}
\]

where \( F_I \) is inertial forces, \( F_\mu \) is friction forces, \( v \) is the wind velocity, \( A \) is the area, \( \mu \) is the dynamic viscosity, \( \partial v / \partial y \) is the velocity gradient near the body with the distance \( y \), \( L \) is the cross-wind dimension of the body, and \( v \) is the kinematic viscosity.

When the kinematic viscosity of air is the same in model and full scale, it would as a consequence of the Reynolds law require extreme model air velocities in order to fulfill the law. For instance, a geometric scale of 1:50 would require 50 times the presumed full-scale wind speed to fulfill the Reynolds law. Normally, this is not realistic and the Reynolds number mismatch may lead to a change of the separation points compared to full scale and hence a significant scaling error in the results. High pressure wind tunnels may overcome this Reynolds number mismatch by reducing the kinematic viscosity of air in model scale.

When a fluid approaches a body, the area around the body becomes a region consisting of disturbed flow. This disturbance in the flow is caused by the separation of the local surface boundary layer at some point along the body. This leads to a change in the flow, going from moving in streamlines in smooth flow into an unsteady flow in different directions. Typically, bodies are distinguished between rounded and sharp-edged bodies. On sharp-edged bodies, the location of the separation point is predefined by the location of the sharp edges while on rounded bodies, which have no sharp edges, the separation point varies depending on the air velocity and thus Reynolds number \( Re \). At \( Re > 40 \) the separation of the boundary layer over the cylinder is
caused by an adverse pressure gradient, which is imposed by the divergent geometry of the flow environment at the rear side of the cylinder. This causes the formation of a shear layer as shown in Figure 2.

Figure 2. Detailed sketch of flow near separation [9]. Reprinted by permission from World Scientific.

For flat plates at zero incidence, the boundary layer is laminar close to the leading edge and becomes turbulent further downstream as the boundary layer thickness increases. At the leading edge there is a constant velocity distribution perpendicular to the plate. As the distance from the leading edge increases, the layer of particles slowed down by the friction becomes larger, as more and more fluid particles are caught up by the retardation, see Figure 3. The transition between laminar to turbulent boundary layer begins with Tollmien-Schlichting (T/S) waves. The T/S waves are weak instabilities in the laminar boundary layer. The growth of the weak instabilities results in non-linear three-dimensional disturbances, which after transform into turbulent spots. The turbulent spots merge giving a fully turbulent boundary layer, and thus completing the transition from a laminar to a turbulent boundary layer.

For a plate with a sharp leading edge, the transition from laminar to turbulent typically takes place at a certain distance $x$ from the leading edge, given by:

$$\text{Re}_{x,\text{crit}} = \left( \frac{v_w x}{v} \right)_{\text{crit}} = 3.5 \cdot 10^5 \text{ to } 10^6$$

Figure 3. Sketch of the transition from laminar to turbulent boundary layer on a flat plate at zero incidence [8]. ① laminar boundary layer. ② unstable Tollmien-Schlichting waves. ③ tridimensional waves and vortex formation. ④ vortex decay. ⑤ formation of turbulent spots. ⑥ fully turbulent boundary layer.

In order to correctly model the load on rounded bodies in model scale, strict fulfilment of Reynolds similarity is required. However, since the location of the separation points for sharp-edged bodies are pre-determined at the edges, the flow field geometry assumes similarity and hence force similarity. Thereby, a relaxation within Reynolds scaling can be allowed for sharp-edged bodies.

On sharp-edged bodies the separation points or lines are defined by the body geometry and widely independent of the actual fluid flow or Reynolds number. Additionally, the reattachment points remain almost correct if the following condition is fulfilled [5]:

$$\text{Re} > \text{Re}_{\text{ons}} = 4000$$

where $\text{Re}_{\text{ons}}$ is the onset Reynolds number for invariant flow field geometry.

As mentioned previously, the subcritical regime is typically the regime, which is reached in low turbulent flow wind tunnel tests. The change of separation point with Reynolds number on rounded bodies therefore implies that forces might be overestimated when no correction is made between model-scale measurements and equivalent estimated full-scale structural properties.
For a bridge cross section, the boundary layer is pretty much similar to a flat plate. The Reynolds number dependency of the reattachment region has been studied in [6] for a slope turning angle of $\alpha = 18^\circ$, see definition in Figure 5. The results showed that the bubble length was reduced with a factor of approx. 10 for Reynolds numbers increasing from approx. $10^4$ to $10^5$. A short bubble is designated as one that only has a local effect on the pressure distribution, whereas a long bubble effects the whole surface resulting in a large reduction in lift and moment.

4 PHYSICS OF VORTEX-INDUCED VIBRATIONS

4.1 Resonance wind velocity

Vortex-induced vibrations may occur when vortices are shed alternately from opposite sides of a structure. This gives rise to a fluctuating load perpendicular to the wind direction. As the vortices are shed alternately from one side and then the other, a harmonically varying cross-wind load with the same frequency as the frequency of the vortex shedding is formed. The shedding frequency $n_s(z)$ of the cross-wind load caused by vortex shedding at location $z$ is

$$n_s(z) = St \frac{v_m(z)}{h(z)}$$  \hspace{1cm} (4)

in which $St$ is the Strouhal number, $v_m$ is the mean velocity of the approaching wind, and $h$ is the cross-wind dimension of the structure considered. The Strouhal number depends on geometry of the structure and on the Reynolds number.

Significant vibrations may occur if the dominating frequency of vortex shedding $n_s$ is the same as the natural frequency $n_e$ for the structure vibrating in a mode in the cross-wind direction. Therefore, the resonance wind velocity $v_r$ is equal to

$$v_m = v_r = \frac{n_s h}{St}$$  \hspace{1cm} (5)

defined by $n_e = n_s$.

4.2 Vortex shedding on a nominally non-vibrating structure

The net load caused by vortex shedding may be expressed by the lift coefficient. This parameter depends on the cross-sectional shape, the Reynolds number, the turbulence scale and intensity, and the aspect ratio.

The net vortex shedding wind load per unit length may be written as

$$F_V(z,t) = q(z)b(z)C_{LV}(z,t)$$  \hspace{1cm} (6)

in which $q(z)$ is the velocity pressure, $b(z)$ is the structural in-wind width, and $C_{LV}(z,t)$ is a non-dimensional lift force coefficient for vortex shedding, where the force has been normalized using the structural in-wind width. It is here chosen to use the in-wind width as structural reference dimension since this is the wind-exposed surface area per unit length for vortex shedding actions.

4.3 Calculation of vortex-induced vibrations

To determine the vibrations of rectangular geometries, the Eurocode use the Scruton number to judge whether vortex-induced vibrations may occur. What is worth noticing in the Scruton number is that the width of the structure is left out, which could be due to the fact that the Eurocode mainly focus on circular cross sections. However, a paper recently published by Svend Ole Hansen [7] shows a modified equation of the Scruton number under the assumption that the aerodynamic damping is proportional to the width of the structure. Therefore, the equation given in [7] accounts for rectangular geometries by including the width $b$. This equation is known as the general mass-damping parameter $S_G$, and is given as

$$S_G = \frac{2\delta_s m_e}{\rho b}$$  \hspace{1cm} (7)

in which $\delta_s$ is the structural damping expressed by the logarithmic decrement, $m_e$ is the effective mass per unit length, and $\rho$ is the air density.

The standard deviation $\sigma_y$ of the structural deflection for a structure with a uniform mode shape may be determined by, see [7]:

$$\frac{\sigma_y}{h} = \frac{1}{St^2} \frac{C_e}{\sqrt{\frac{\rho b h}{m_e} \sqrt{\left( \frac{\rho b h}{m_e} \right)^2 + \frac{4\pi}{4\pi - K_{DG}} \left( 1 - \left( \frac{\sigma_y}{a_1 h} \right)^2 \right)}}}$$  \hspace{1cm} (8)
in which \( l \) is the length of the structure and \( C_c, a_L, \) and \( K_{aG} \) are aerodynamic parameters governing the response of the structure as illustrated in Figure 4.

![Figure 4. Physical interpretation of the aerodynamic parameters in Equation (8).](image)

The aerodynamic parameter \( C_c \) governs the standard deviation of the deflection when the lift forces are dominating. The limiting factor \( a_L \) defines the normalized standard deviation of cross-wind deflection \( \frac{\sigma_y}{h} \) for a general mass-damping parameter of 0, see Figure 4. The proportionality damping factor \( K_{aG} \) times \( 4\pi \) locates the transition range at which the smaller deflections governed by lift forces change to larger deflections governed by motion-induced forces, see Figure 4.

5 BRIDGE DECKS

Three streamlined steel box girders shown in Figure 5 have been investigated with two main focus areas:

- Influence of the down- and upstream slope turning angle \( \alpha \), shown in Figure 5, on vortex-induced vibrations. The three cross sections have the slope turning angles \( \alpha \) of 11.8°, 15.8°, and 19.8°.
- Influence of the Reynolds number on the forces and vibration amplitudes.

The Reynolds number at resonance is defined as

\[
Re = \frac{v_r \cdot h}{\nu}
\]

where \( h \) is the cross-wind height of the bridge deck, see Figure 4, and \( \nu \) is the kinematic viscosity of air at 25 °C, at which the tests have been conducted at, is \( 1.65 \cdot 10^{-5} \) m\(^2\)/s.

The three cross sections have been tested on scale 1:50 and 1:10 section models. The bridge decks are modelled as stiff sections with a low mass, allowing extra mass to be added when mass scaling and natural frequencies are set. The surface of the 1:50 model is made of foam, while the surface of the 1:10 model is sanded epoxy with a layer of paint, giving a surface roughness comparable to a painted wall for both models. All cross sections have been tested without any type of railings, wind shields or other equipment. Initial tests with end plates did not show any difference from initial tests without end plates. End plates have therefore not been used in the results shown.

![Figure 5. Streamlined single box girders. Dimensions are in full scale [m]. The slope turning angle \( \alpha \) is defined for section 1a.](image)

All tests were performed in low turbulent flow with a turbulence intensity below approximately 1%. Drag wires were used in order to stabilize the section models in the in-wind direction. The 1:50 and 1:10 models may be seen in Figure 6 and Figure 7, respectively.

The bridge decks considered in the present study all have large cross section width compared to height, indicating that they may have significant Reynolds number effects. This is attributed to the separation at the upstream corner at the bottom surface, the reattachment of the flow to the bottom surface, and subsequently the separation at the downstream trailing edge at the bottom surface. The separation and reattachment phenomena are known to be closely related to Reynolds number sensitivity, see the theoretical evaluations in chapter 3.
INVESTIGATION RESULTS OF MODEL 1b

The present chapter will present the wind tunnel investigation results for model 1b and full-scale results.

6.1 Nominally non-vibrating setup and full-scale results

In order to investigate the Reynolds number effect on cross section 1b, a series of nominally non-vibrating model measurements were performed on the scale models 1:50 and 1:10. Six different Reynolds numbers were investigated for both scale models. The results are presented in terms for spectral power of $C_{LV}$, see Equation (6), in Figure 8 for the 1:50 model and Figure 9 through Figure 11 for the 1:10 model.

Figure 8 and Figure 9 show the spectral power of $C_{LV}$ in relation to the frequency $n$ for the 1:50 and 1:10 model, respectively. In Figure 8, two peaks are observed for each Reynolds number, namely a peak due to the natural frequency of the test setup and a peak due to vortex shedding occurring at different frequencies due to the different tested wind velocities. Likewise, two peaks are observed in Figure 9 for each Reynolds number, namely a peak due to the model’s natural frequency and a peak due to vortex shedding occurring at different frequencies.

A close-up of the vortex shedding peaks in Figure 9 may be viewed in Figure 10 and Figure 11, which show the spectral power of $C_{LV}$ in relation to the Strouhal number for the lowest and highest three tested Reynolds numbers, respectively. The bending frequency of the section model has been eliminated by applying a frequency response function.
Two main observations are made in Figure 10 and Figure 11. Firstly, the amount of energy contribution from the vortex shedding at resonance, i.e. the width of the peak, varies depending on the Reynolds number. The standard deviation within ±20% from the Strouhal number generally decreases with increasing Reynolds number, see Figure 12. The standard deviation at Re = 137·10^3 has not been plotted, as the standard deviation at Re = 137·10^3 will be highly influenced by resonance between the vortex shedding frequency and the model’s first bending frequency as seen in Figure 9. Resonance between the model’s first bending frequency and the vortex shedding frequency may also slightly influence the standard deviation at Re = 91.5·10^3.

Secondly, the Strouhal number displays a significant dependence on the Reynolds number showing an increasing $St$ with increasing Re, see also Figure 17.

A test program involving simultaneous recording of fluctuating wind pressure on the box girder surface, wind flow velocities, and structural accelerations has been carried out on Gjemnessund Suspension Bridge [10]. Fluctuating pressure on the box girder surface is recorded by pressure transducers connected to pressure taps on the surface by pneumatic tubes. Figure 13 shows the normalized spectrum of lift load, where the calculation is performed by dividing the time series of 90 seconds into eight parts with equal length, where each part overlaps half the neighbour on both sides effectively smoothing the graph of the estimated spectrum.

In Figure 13, the mean wind velocity component perpendicular to the bridge axis is 2.7 m/s and the mean wind velocity component along the bridge is 4.6 m/s, giving a yaw angle of 60° for the mean wind velocity. At such low mean wind velocity turbulence may suppress much of the development of a distinct vortex shedding load process.

For the full-scale measurements shown in Figure 13, the Reynolds number is approximately 900·10^3. Spectral distributions of the lift forces measured in the wind tunnel have the same general shape as found in full-scale pressure measurement on a bridge deck, and the general shape is in agreement with the theoretical spectral model of vortex-induced forces on structures.
Figure 12. Standard deviations based on the normalized spectral lift force contributions, $C_{LV}$ in Equation (6), with frequencies being within ±20% from the Strouhal number for five tested Reynolds numbers on the 1:10 model, where the resonance frequency is not coinciding with the bending frequency of the model.

Figure 13. Normalized spectrum of lift force measured in full scale on Gjemnessund Suspension Bridge [10] with a height of $h = 2.6$ m. The wind approaches the bridge deck with a yaw angle of 60° and with a mean wind velocity component of $v_m = 2.7$ m/s perpendicular to the bridge axis. Re is approximately $900 \cdot 10^3$ based on the skew wind speed of 5.3 m/s.

A comparison of the Strouhal number with the drag coefficient for the 1:10 model is shown in Figure 14 as function of Re. $St$ is found by the power spectral densities of the fluctuating aerodynamic lift forces for different values of wind tunnel wind velocity.

Like $St$, $C_D$ displays similar dependence on Re. Although, $C_D$ decreases with increasing Re which is somewhat similar to a circular cylinder. The change in $St$ and $C_D$ with increasing Re clearly indicates a change in the topology of the flow probably linked to the location of separation and reattachment zones. This behavior is typically not expected for flow around sharp edged bodies. At low Reynolds numbers, the test results suggest that this separation generates larger vortex-induced vibrations at lower Reynolds numbers. In [6], it is described how the bubble length is reduced with increasing Re. For the present streamlined bridge deck sections, the reduction in bubble length may be crucial for the susceptibility to vortex-induced vibrations. Larger bubble lengths at lower Reynolds numbers may increase the loading area with pressure fluctuations close to the Strouhal frequency, indicating larger response when resonance with the natural frequency occurs.

The Reynolds number dependence of Strouhal number and drag coefficient shown in Figure 14 indicates that $St$ could be inversely proportional to the width of the section wake and thus to $C_D$, hence $St \cdot C_D$ approximately equals a constant. Similar behavior has been observed on the approach spans of the Great Belt Bridge [12].

Figure 14. Drag force coefficients, $C_D = F_D/(q_o b^2 l)$, and Strouhal numbers for different Reynolds numbers for cross section 1b determined in the nominally non-vibrating setup. Below Re = $4 \cdot 10^4$ for the 1:50 model and above Re = $4 \cdot 10^4$ for the 1:10 model.
The dependency of $C_D$ and $St$ with Reynolds number indicate that the pressure distribution around the bridge sections will also be dependent on Reynolds number.

### 6.2 Dynamic setup

The results from the nominally non-vibrating setup clearly showed a dependency on Reynolds number of the Strouhal number and drag coefficient. In order to investigate this further, a series of dynamic measurements have been performed at different Reynolds numbers and mass-damping parameters for the 1:50 and 1:10 model.

Figure 15 shows the vortex-induced vibrations measured in the wind tunnel on cross section 1b at the resonance wind velocity after build-up for several mass-damping parameters. For two Reynolds numbers, a prediction based on Equation (8) has been fitted. The influence of the general mass-damping parameter on the vibration amplitudes is seen to, as expected, result in smaller vibration amplitudes as the mass-damping parameter is increased for both the 1:50 model and the 1:10 model. Like the results from the nominally non-vibrating setup, the dynamic results show a Reynolds number dependency in terms of a decreasing vibration amplitude with increasing Reynolds number. For $Sc_G \approx 4$, this is clearly illustrated in Figure 16 where the vibration amplitude is seen to decrease for increasing Reynolds number.

Experience shows that streamlined bridge decks with generalized mass-damping parameters $Sc_G$ of less than approx. 6 may be susceptible to vortex-induced vibrations if no guide vanes are added on the cross section. This is also predicted by model tests at low Reynolds numbers of approx. 5-8,000, but not at intermediate Reynolds number 20-70,000, see Figure 15 and Figure 16. However, whether it is the scale 1:50 section models or 1:10 section models that most accurately resemble a full-scale 1:1 bridge section remain unanswered. Full-scale measurements on the existing Great Belt Suspension Bridge [13] do show a good correlation with wind tunnel tests on scale 1:50 section models, indicating that the vibration amplitudes may drop at an intermediate Reynolds number range, and this is similar to the behaviour of circular cylinders.

![Figure 15](image1.png)

**Figure 15.** Measurements at several $Sc_G$ and predictions based on Equation (8) with $St = 0.25$, $\alpha_L = 0.017$, $K_dG = 0.55$, and $C_c = 0.00083$ for the 1:50 model and $St = 0.32$, $\alpha_L = 0.010$, $K_dG = 0.13$, and $C_c = 0.00030$ for the 1:10 model.

![Figure 16](image2.png)

**Figure 16.** Measurements at $Sc_G \approx 4$ showing the vibration amplitude dependency of $Re$. The 1:50 model and 1:10 model wind velocities are between 1.10-1.94 m/s and 1.43-3.63 m/s, respectively.

### 6.3 Strouhal number

The Strouhal number may be found by lift force spectra determined in the nominally non-vibrating setup and also by the vibration frequency in the dynamic setup. The Strouhal number for the 1:50 model was found to be 0.25 while the Strouhal number for the 1:10 model was found in the range of 0.32 to 0.35, both in the dynamic rig. The Strouhal numbers found in the nominally non-vibrating tests are approximately 0.30 for the 1:50 model and between 0.40 and 0.47 for the 1:10 model depending on the Reynolds number. A comparison between the Strouhal numbers found in the nominally non-vibrating and dynamic setup may be seen in Figure 17. For comparison a similar illustration for circular cylinders is shown in Figure 18.

The change of the bridge Strouhal numbers for vibrating models clearly show a lock-in phenomenon similar to the results found for circular cylinders. This observation is due to the difference in flow separations for the two situations.

At Reynolds numbers just below 30,000, the nominally static and dynamic Strouhal numbers are identical, while for Reynolds numbers of approximately 70,000 a difference between the nominally static and dynamic Strouhal numbers is observed. Above a certain critical Reynolds number in between 30,000 and 70,000, the Strouhal number becomes dependent on the motion of the bridge deck, and thus gives different Strouhal numbers for the two setups. This behaviour is also observed for circular cylinders in Figure 18, however at a different critical Reynolds number.
Figure 17. Comparison of the Strouhal number determined in nominally non-vibrating and dynamic setups.

Figure 18. Strouhal number as function of Reynolds number [2].

At low Reynolds numbers, i.e. Re lower than approximately 10,000-15,000, vortex shedding was not found in the lift force spectra determined in the nominally non-vibrating setup. This could indicate that the onset Reynolds number for invariant flow field geometry of 4000 given in [5] is not fully applicable for streamlined bridge decks in a nominally non-vibrating setup. However, in the dynamic rig vortex-induced vibrations were observed for Reynolds numbers as low as approximately 4000, yet again proving a difference in the flow separations between the nominally static setup and the dynamic rig.

7 INVESTIGATION RESULTS OF MODELS 1a, 1b, AND 1c

The present chapter deals with the influence of downstream and upstream slope turning angle on vortex-induced vibrations for model 1a to 1c.

Figure 19 shows the vibration amplitude at several mass-damping parameters at Re ≈ 75·10^3 for 1a to 1c on scale 1:10 models. It is seen that the slope turning angle α has a significant influence on the vibration amplitude for vortex-induced vibrations. At ScG ≈ 2, the vortex-induced vibrations are reduced as α becomes smaller, going from approx. σy/h = 0.016 to σy/h = 0.006 when α is reduced from 19.8° to 15.8°, and further down to σy/h = 0.000 when α is reduced to 11.8°. As for the 1:10 model, the 1:50 model of 1a shows no sign of vortex-induced vibrations.

For the 1:50 model of 1b and 1c, the influence of α is not evidently clear, see Figure 20. This may be caused by the high dependency on especially low Reynolds numbers that was observed previously in Figure 16, thus complicating a direct comparison between model 1b and 1c as the cross sections may have different degrees of dependency on Reynolds number. What is evident is that the vibration amplitudes for 1b at Re = 4.32·10^3 lie within σy/h = 0.018 and σy/h = 0.011, meaning the 1:50 model 1b could be either higher or lower than the 1:50 model 1c, which has vibration amplitudes of σy/h = 0.017 at Re = 4.32·10^3.

The effect of the slope turning angle has also been considered in [16], recommending a maximum angle of 15° in order to avoid vortex-induced vibrations. According to the present measurements, significant vibrations were experienced for an angle of 15.8°, and an angle of 11.8° was found to remove the tendency for vortex-induced vibrations. Thus, the exact limit angle for insignificant vibrations seems to be debatable.

The prediction for 1c is, as the predictions for 1b for the 1:50 and 1:10 models, seen to fit the measurements surprisingly well. This indicates that Equation (8), which was originally proposed for circular cylinders and later adopted to rectangular cross sections in [7], may also provide useful predictions for bridge deck cross sections.
Figure 19. Amplitude comparison between the 1:10 model of 1a, 1b, and 1c at \( Re \approx 75 \cdot 10^3 \). The predictions for 1b and 1c are based on Equation (8) with \( St = 0.32, a_L = 0.010, K_{ao} = 0.13, \) and \( C_c = 0.00030 \) for the 1b 1:10 model and \( St = 0.29, a_L = 0.019, K_{ao} = 0.26, \) and \( C_c = 0.00006 \) for the 1c 1:10 model.

Figure 20. Amplitude comparison between the 1:50 model of 1a, 1b, and 1c at \( Re \approx 4.75 \cdot 10^3 \).

8 REMEDIES FOR STREAMLINED BOX GIRDER BRIDGE SECTION

For Hardanger Suspension Bridge described in [15], the general non-dimensional mass-damping parameter \( Sc_G \) is equal to approximately 6.6 for a structural logarithmic damping decrement \( \delta_s \) assumed to be 2%. This indicates that the bridge may be susceptible to non-acceptable vortex-induced vertical vibrations, and this was confirmed in initial section model tests in a wind tunnel with the bridge deck without any geometrical mitigation devices installed. Subsequently, a series of tests have been carried out with vortex mitigation devices in form of guide vanes and a vortex spoiler, see Figure 21 and Figure 22. A first set of dynamic tests showed that guide vanes at the bottom part of the cross section close to the bends could suppress the torsional vibrations of the bridge section to a low level. Many different guide vanes were tested, however, none of the guide vanes were able to suppress the vortex-induced vertical vibrations of the bridge section to a sufficiently low level when the simulated structural damping was low.

Figure 21. Streamlined cross section of the Hardanger Suspension Bridge. Dimensions are in full scale [m].

Figure 22. Left photo: bottom of the section model. Right photo: top of the section model.

An extensive study was initiated in order to improve the dynamic behaviour of the section model. This study led to a design of vortex mitigation devices in form of guide vanes and a centrally located vortex spoiler that efficiently reduces the vortex-
9 CONCLUSION

The following may be concluded for the present streamlined bridge decks:

- The spectral distributions of the lift forces measured in the wind tunnel have the same general shape as found in full-scale pressure measurement on a bridge deck, and the general shape is in agreement with the theoretical spectral model of vortex-induced forces on structures.
- Theoretical fit of vortex-induced vibration amplitudes based on the spectral model is in good agreement with the measurements.
- The resonance wind velocities for vortex-induced vibrations are governed by the Strouhal number, and the Strouhal number should be based on results from vibrating structures. At certain Reynolds number ranges, nominally non-vibrating structures seem to overestimate the Strouhal number relevant for vortex-induced vibrations.
- The Strouhal number increases with the Reynolds number, where the drag force coefficient decreases. This opposite Reynolds number dependency is in agreement with results obtained from other structures.
- The vortex-induced vibrations depend strongly on the Reynolds number.
- Typical low and intermediate wind tunnel ranges of the Reynolds number have been investigated. However, it is not easy to extrapolate the results to the actual vortex-induced vibrations of the full-scale structures with their larger Reynolds numbers.
- Full-scale experience agrees reasonably well with section model test results carried out at low Reynolds numbers, and it is not obvious that intermediate Reynolds numbers give vortex-induced vibrations closer to the full-scale behaviour.
- For all tested Reynolds numbers, the vortex-induced vibrations measured are less significant for the profile with the smallest slope turning angle $\alpha$ of 11.8°. For the larger slope turning angles tested, $\alpha$ equal to 15.8° and 19.8°, the most susceptible profile to vortex-induced vibrations depends on the Reynolds number.
- Vortex mitigation devices in form of guide vanes and a vortex spoiler have proven to efficiently reduce vortex-induced vibrations.

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CONCLUSIONS

The guide vanes are located at the outer lower edges of the bridge section and the vortex spoiler is located centrally on the bottom face of the bridge section, see Figure 21 and Figure 22.

For low turbulent flow and low structural damping with logarithmic damping decrement of approximately 0.5%, the vertical vibration amplitude was reduced from approximately 15% of the deck height to approximately 11% with guide vanes only and further reduced to approximately 3% by the centrally located vortex spoiler, see Figure 21 and Figure 22. Thus, the centrally located vortex spoiler have shown to be an effective mean for suppressing vortex-induced vibrations of bridge decks already equipped with guide vanes.


